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Convolved numbers of *k*-section of the Fibonacci sequence: properties, consequences

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ABSTRACT

Chebyshev polynomials are used to solve numerous applied problems in computer science involving interpolation theory, approximation theory, numerical analysis, dynamical systems theory, and number theory. However, when forming pseudo-random recurrent sequences, the use of derivatives of Chebyshev polynomials, especially of higher orders, is much less common in the literature, **although the issue is quite actual**. This article somewhat fills this gap. It is shown that among recurrent sequences used for information analysis and to improve its cryptographic protection, the Fibonacci sequence and its generalizations are the most popular. This article considers a further generalization of Fibonacci numbers, namely, folded numbers *k*-sections of the Fibonacci sequence. **The objective of the research** is to further generalize the Fibonacci numbers, namely, the collapsed numbers of the k-intersection of the Fibonacci sequence. **The research used modern methods** of number theory. The properties of the obtained sequences are determined, and new connections between their elements are found. A further development of Fibonacci-type sequences is proposed, based on the relationship between the derivatives of Chebyshev polynomials of the second kind and Chebyshev polynomials themselves, as well as on the relationship between the folded numbers – the cross-sections of *k* the Fibonacci sequence and the derivatives of Chebyshev polynomials of the second kind through Lucas numbers. A number of identities are obtained linking Fibonacci numbers and Lucas numbers. It is shown that higher-order derivatives of Chebyshev polynomials prove to be an effective basis for solving certain problems in number theory, namely, the construction of new sequences. **Thus, the study resulted** in a family of generalized sequences with higher order growth and more complex coupling coefficients than the known Fibonacci sequences. This makes the resulting generalized sequences ideal for sparse data compression and for solving a number of problems in information security. It is proven that the resulting sequences are original and are not presented in the OEIS encyclopedia, confirming the potential of the proposed approach to the formation of various sequences that can be used to improve the reliability of information systems.

Keywords: Information systems; data encryption; linear shift algorithm; Fibonacci numbers; Lucas numbers; folded Fibonacci numbers; *k*-sections of the Fibonacci sequence; derivatives of Chebyshev polynomials of the second kind

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INTRODUCTION

The widespread use of the internet and other public computer networks is increasingly facilitating the globalization of electronic information exchange in the socio-political, economic, financial, medical, and military spheres. However, with the development of these systems, the vulnerability of the information space increases, creating favorable conditions for interception, destruction, falsification, and other illegal actions by attackers against electronic documents of various formats.

One of the main problems of digital data processing is the problem of unauthorized access to information. To protect data in the digital environment cryptography methods are used. Cryptography is an integral part of modern society (electronic mail, online shopping, health information, block-chain technologies etc.).

There are different data encryption schemes; one of the possible schemes is related to stream ciphers, which use a sufficiently long pseudo-random sequence. A stream scheme converts a stream of text characters into a stream of cipher-text, and the conversion depends on the state of the system. Identical text characters will be encrypted into different ciphertext characters. The simplest stream scheme is the Vernam cipher [1].

To increase the cryptographic strength of the cipher, linear or nonlinear shift algorithms are additionally used. Linear shift gives speed, nonlinear increases cryptographic strength. If a pseudo-random sequence is generated using nonlinear maps, then it is possible to additionally use a linear shift. Such shifts are generated by linear recurrent sequences. Among them, the most popular are the *Fibonacci* sequence and its generalizations [2]. On various aspects of cryptography there exist numerous literatures, for example, [3], [4].

The connection between combinatorial methods inherent in the properties of recurrent sequences and the problem of cryptanalysis once again demonstrates the philosophy of intertwining different branches of mathematics, how from one fascinating theoretical problem solutions to other quite practical problems can be generated.

PROBLEM STATEMENT

The *Fibonacci* sequence is defined by the recurrence relations

$$x_{n+2} = x_{n+1} + x_n, \;\; n = 1, 2, \dots \; , \qquad (1)$$

and initial conditions $x_1 = 1$, $x_2 = 1$.

In turn, the *Lucas* sequence is defined by the same relations and initial conditions $x_1 = 1$, $x_2 = 3$ ..

According to the OEIS electronic encyclopedia [5] these sequences have their own numbers: for the *Fibonacci* sequence $\{F_n\}_{n=1}^{\infty} = \{1, 1, 2, 3, 5, 8, 13, 21, 34, 55, \ldots\}$ the number is A000045; for the *Lucas* sequence $\{L_n\}_{n=1}^{\infty} = \{1, 3, 4, 7, 11, 18, 29, 47, 76, 123, \ldots\}$ it is A000032 respectively.

Other examples of linear recurrence sequences of the second order with different initial conditions can be found in [6]. For example, the recurrence sequence

$$x_{n+2} = 3x_{n+1} - x_n, \;\; n = 1, 2, \dots \; ,$$

with the initial conditions $x_1 = 1$ and $x_2 = 3$, defines a section of the Fibonacci sequence $\{F_{2n}\}_{n=1}^{\infty} = \{1, 3, 8, 21, 55, \ldots\}$ with OEIS number A001906, while with the initial conditions $x_1 = 1$, $x_2 = 2$ defines the section $\{F_{2n-1}\}_{n=1}^{\infty} = \{1, 2, 5, 13, 34, 89, \ldots\}$ with OEIS number A001519.

Let us look at another example. Denote $\Phi_{n,k} = F_{nk}/F_k$, $n = 1, 2, \dots$, $k = 1, 2, \dots$. Since the number nk is a multiple of k , then all numbers $\Phi_{n,k}$ are integers [7]. In addition, for each $k = 1, 2, \dots$ the sequence $\{\Phi_{n,k}\}_{n=1}^{\infty}$ is defined by the recurrence relation

$$x_{n+2} = L_k x_{n+1} - (-1)^k x_n, \;\; n = 1, 2, \dots \; , \qquad (2)$$

and initial conditions $x_1 = 1$, $x_2 = L_k$ [7, 8].

The numbers $\Phi_{n,k}$ can be expressed directly via *Lucas* numbers [9]

$$\Phi_{n,k} = \sum_{j=0}^{\left[\frac{n-1}{2}\right]} (-1)^{(k-1)j} \binom{n-1-j}{j} (L_k)^{n-1-2j} . \qquad (3)$$

For each k , the sequence $\{\Phi_{nk}\}_{n=1}^{\infty}$ has its own number in [5], however, it does not have a special name (at least the authors did not find this name in the literature). In the present article, we will call this sequence as k **-section of the *Fibonacci* sequence.** Corresponding numbers in OEIS are: $k = 1$, A000045 (*Fibonacci* sequence), $k = 2$, A001906 (section of the *Fibonacci* sequence); $k = 3$, A001076; $k = 4$, A004187; $k = 5$, A049666; $k = 6$, A049660; $k = 7$, A049667; $k = 8$, A049668; $k = 9$, A049669, etc.

Note that numbers generated by general second-order recurrence relations $x_{n+2} = \alpha x_{n+1} \pm x_n, \;\; n = 1, 2, \dots$, are studied sufficiently enough, however specifically the sequence $\{\Phi_{n,k}\}_{n=1}^{\infty}$ is generated by «the golden ratio» (expressed via numbers $\varphi = \frac{1}{2}\left(1 + \sqrt{5}\right)$, $\varphi^{-1} = \frac{1}{2}\left(-1 + \sqrt{5}\right)$.

The next generalization of the Fibonacci sequence is related to convolved numbers.

Convolved *Fibonacci* numbers are defined as follows:

$$F_n^{(0)} = F_n , \;\; F_n^{(s)} = \sum_{j=0}^{n-1} F_{j+1} F_{n-j}^{(s-1)} , \;\; s = 1, 2, 3, \dots \; . \qquad (4)$$

From formulas (4) it follows that $F_1^{(s)} = 1$, $F_2^{(s)} = s + 1$. When defining convolved numbers, also modification of formulas (4) (simple shift by indices), where $F_{s+1}^{(s)} = 1$, $F_{s+2}^{(s)} = s + 1$ and $F_n^{(s)} = 0$ for $n < s$ [10]. It seemed to us that the form of defining convolved numbers via (4) is more convenient, and such a form will be used in the article below.

For each s, the sequence $\left\{F_n^{(s)}\right\}_{n=1}^{\infty}$ has its own OEIS number:

$\left\{F_n^{(1)}\right\}_{n=1}^{\infty}=\{1,2,5,10,20,38,\dots\}$ – A001629;

$\left\{F_n^{(2)}\right\}_{n=1}^{\infty}=\{1,3,9,22,51,111,\dots\}$ – A001628;

$\left\{F_n^{(3)}\right\}_{n=1}^{\infty}=\{1,4,14,40,105,256,\dots\}$ – A001872;

$\left\{F_n^{(4)}\right\}_{n=1}^{\infty}=\{1,5,20,65,190,511,\dots\}$ – A001873

etc.

Convolved *Fibonacci* numbers were studied in several papers - see some references, for example, in [5]. The sequence $\left\{F_n^{(s)}\right\}_{n=1}^{\infty}$ is defined by a linear recurrence relation of order higher than the second (for most s the order is equal to $2s$).

For example, for $s=1$ convolved numbers satisfy the relation [10]:

$$x_{n+4}=2x_{n+3}+x_{n+2}-2x_{n+1}-x_n,\ \ n=1,2,\dots\ .$$

These recurrence relations are easy to obtain by the method of generating functions (this method will be considered further). The connection between generating functions and recurrence relations is described in detail, for example, in [7], [11].

For numbers from k -sections of the *Fibonacci* sequence it is also possible to define convolved numbers by a formula similar to (4), namely

$$\begin{aligned}\Phi_{n,k}^{(1)}&=\sum_{j=0}^{n-1}\Phi_{j+1,k}\Phi_{n-j,k}\,,\\ \Phi_{n,k}^{(s)}&=\sum_{j=0}^{n-1}\Phi_{j+1,k}\Phi_{n-j,k}^{(s-1)}\,,\ s=2,3,\dots\ .\end{aligned}\tag{5}$$

Since $\Phi_{n,1}=F_n$, then $\Phi_{n,1}^{(s)}=F_n^{(s)}$.

The main goals of the presented work are:

– finding representation formulas of convolved numbers k -sections of the *Fibonacci* sequence;

– determination of properties of the sequence $\left\{\Phi_{n,k}^{(s)}\right\}_{n=1}^{\infty}$, finding new connections between elements of the sequence;

– finding of *Binet* type formulas for convolved numbers $\Phi_{n,k}^{(s)}$.

To achieve these goals, the formulas connecting derivatives of the Chebyshev polynomials of the second kind and the Chebyshev polynomials of the second kind themselves were used [12]. As side results, various identities related to the elements of *Fibonacci* and *Lucas* numbers, binomial coefficients, convolved *Fibonacci* numbers and numbers k -sections of the *Fibonacci* sequence were obtained.

Additionally, we note that the sequences $\left\{\Phi_{n,k}^{(s)}\right\}_{n=1}^{\infty}$ for $k=3,4,\dots,\ s=1,2,\dots$ are not represented at the OEIS encyclopedia [5].

PRELIMINARY RESULTS

Relation (1) can be considered as a linear difference equation with the given initial conditions. The roots of the corresponding characteristic equation are φ and $-\varphi^{-1}$, and the particular solution is represented in the form

$$F_n=\frac{\varphi^n-(-\varphi)^{-n}}{\varphi+\varphi^{-1}}=\frac{1}{\sqrt{5}}\left(\varphi^n-(-\varphi)^{-n}\right). \tag{6}$$

Similarly,

$$L_n=\varphi^n+(-\varphi)^{-n}. \tag{7}$$

Formulas (6), (7) are called *Binet* formulas for *Fibonacci* and *Lucas* numbers [7].

The function $f(z)=\dfrac{z}{1-z-z^2}$ is the generating function for the *Fibonacci* sequence [7]; this means that

$$\frac{z}{1-z-z^2}=\sum_{j=1}^{\infty}F_j\,z^j\ .$$

For the section of the *Fibonacci* sequence $\left\{F_{2n}\right\}_{n=1}^{\infty}$ the generating function will be $\hat{f}(z)=\dfrac{z}{1-3z+z^2}$, where $\dfrac{z}{1-3z+z^2}=\sum\limits_{j=1}^{\infty}F_{2j}\,z^j$.

Consider k -section of the *Fibonacci* sequence. Using (2) and the connection between the generating function and recurrence relations, one can determine the generating function for $\Phi_{j,k}$ [7, p.230]:

$$\hat{f}_k(z)=\frac{z}{1-L_kz+(-1)^kz^2}\,,$$

where $\dfrac{z}{1-L_kz+(-1)^kz^2}=\sum\limits_{j=1}^{\infty}\Phi_{j,k}\,z^j$ ($\hat{f}_0(z)=f(z)$, $\hat{f}_1(z)=\hat{f}(z)$).

For convolved *Fibonacci* numbers $F_n^{(s)}$ the generating function is defined as [10] $\tilde{f}(z)=\frac{z}{\left(1-z-z^2\right)^{s+1}}$; $\frac{z}{\left(1-z-z^2\right)^{s+1}}=\sum_{j=1}^{\infty}F_j^{(s)}z^j$.

The function $g(z,t)=\frac{1}{1-2tz+z^2}$ is called the generating function for Chebyshev polynomials of the second kind $U_n(t)$ [13], so that

$$\frac{1}{1-2tz+z^2}=\sum_{j=0}^{\infty}U_j(t)z^j,$$

where [14] $U_n(t)=\sum_{j=0}^{\left[\frac{n}{2}\right]}(-1)^j\binom{n-j}{j}2^{n-2j}t^{n-2j}$.

Since $zg(z/i,i/2)=f(z)$, $zg(z,3/2)=\hat{f}(z)$ ($i^2=-1$), then [7]

$$(-i)^{n-1}U_{n-1}(i/2)=F_n, \tag{8}$$

$$U_{n-1}(3/2)=F_{2n}. \tag{9}$$

Further, $zg(z/i,(i/2)L_k)=\hat{f}_k(z)$ if k is odd, and $zg(z,(1/2)L_k)=\hat{f}_k(z)$, if k is even. Then

$$\Phi_{n,k}=\frac{F_{nk}}{F_k}=\begin{cases}(-i)^{n-1}U_{n-1}\left(\frac{i}{2}L_k\right), k-odd,\\ U_{n-1}\left(\frac{1}{2}L_k\right), k-even\end{cases} \tag{10}$$

Let us return to convolved *Fibonacci* numbers $F_n^{(s)}$. Let us find

$$\frac{\partial^s}{\partial t^s}g(z,t)=2^s s!\frac{z^s}{\left(1-2tz+z^2\right)^{s+1}}=$$
$$=\sum_{j=0}^{\infty}U_j^{(s)}(t)z^j=\sum_{j=s}^{\infty}U_j^{(s)}(t)z^j=$$
$$=\sum_{j=1}^{\infty}U_{j+s-1}^{(s)}(t)z^{j+s-1}$$

($U_j^{(s)}(t)=0$, $j<s$).

Then $\frac{z}{\left(1-2tz+z^2\right)^{s+1}}=\frac{1}{2^s s!}\sum_{j=1}^{\infty}U_{j+s-1}^{(s)}(t)z^j$, consequently,

$$F_n^{(s)}=\frac{(-i)^{n-1}}{2^s s!}U_{n+s-1}^{(s)}(i/2). \tag{11}$$

Similarly,

$$F_{2n}^{(s)}=\frac{1}{2^s s!}U_{n+s-1}^{(s)}(3/2). \tag{12}$$

Formulas (10), (11), (12) are a generalization of the well-known formulas (8) and (9).

MAIN RESULTS

Further results essentially are obtained on the formulas connected derivatives of Chebyshev polynomials of the second kind $U_n^{(s)}(z)$ (s is the order of the derivative) and the Chebyshev polynomials of the second kind themselves $U_n(z)$. Let us present these formulas [12].

$$U_n^{(s)}(z)=s!\sum_{j=0}^{\left[\frac{n-s}{2}\right]}(-1)^j\binom{n-j}{j}\binom{n-2j}{s}2^{n-2j}z^{n-2j-s}, \tag{13}$$

$$\begin{aligned}&\frac{1}{2^s s!}U_{n+s-1}^{(s)}\left(\frac{1}{2}\left(z^{\frac{1}{2}}+z^{-\frac{1}{2}}\right)\right)=\\&=z^{-\frac{n-1}{2}}\sum_{j=0}^{n-1}\binom{n+s-1-j}{s}\binom{s+j}{s}z^j\end{aligned}, \tag{14}$$

$$\begin{aligned}&\frac{1}{2^s s!}U_{n+s-1}^{(s)}\left(\frac{1}{2}\left(z^{\frac{1}{2}}+z^{-\frac{1}{2}}\right)\right)=\\&=z^{-\frac{n-1}{2}}(1-z)^{-(2s+1)}\cdot\\&\cdot\sum_{j=0}^{s}(-1)^j\binom{n+2s}{j}\binom{n+s-1-j}{n-1}\left(z^j-z^{n+2s-j}\right)\end{aligned}, \tag{15}$$

$$\begin{aligned}&\frac{2^s}{s!}\left(1-z^2\right)^sU_{n+s-1}^{(s)}(z)=\\&=(-1)^s\sum_{j=0}^{s}(-1)^j\binom{n+s-1-j}{n-1}\binom{n+2s}{j}U_{n+2s-1-2j}(z)\end{aligned}. \tag{16}$$

Binet type formulas for convolved *Fibonacci* numbers are obtained from (11), (14), (15) by substituting $-\varphi^2$ for z:

$$\begin{aligned}&F_n^{(s)}=(-1)^{n-1}\varphi^{-n+1}\cdot\\&\cdot\sum_{j=0}^{n-1}(-1)^j\binom{n+s-1-j}{s}\binom{s+j}{s}\varphi^{2j},\end{aligned}$$

$$\begin{aligned}&F_n^{(s)}=\frac{\varphi^{-n+1}}{\left(1+\varphi^2\right)^{2s+1}}\sum_{j=0}^{s}(\binom{n+2s}{j}\binom{n+s-1-j}{n-1}\cdot\\&\cdot\left(-(-1)^n\varphi^{2j}+\varphi^{2(n+2s-j)}\right))\end{aligned}. \tag{17}$$

From formulas (17) and (6) follow the connection between convolved *Fibonacci* numbers and *Fibonacci* numbers:

$$F_n^{(s)} = 5^{-s}\sum_{j=0}^{s}\binom{n+2s}{j}\binom{n+s-1-j}{n-1}F_{n+2s-2j} \,. \quad (18)$$

In derivation of (18), the formulas $\frac{\varphi^{-n+1}}{\left(1+\varphi^2\right)^{2s+1}} = \frac{\varphi^{-n}\varphi^{-s}}{\left(\varphi+\varphi^{-1}\right)^{2s}\left(\varphi+\varphi^{-1}\right)}$, and $\frac{-(-1)^n\varphi^{-(n+2s-2j)}+\varphi^{n+2s-2j}}{\varphi+\varphi^{-1}} = F_{n+2s-2j}$ were used.

Formula (18) can also be obtained from (16) by substituting $z = i/2$. This formula substantially refines formula (1.12) from [10].

For $s=1$: $F_n^{(1)} = \frac{1}{5}\left(nF_{n+2}+(n+2)F_n\right)$.

For $s=2$:

$$F_n^{(2)} = \frac{1}{25}\cdot$$
$$\cdot\left(\frac{1}{2}n(n+1)F_{n+4}+n(n+4)F_{n+2}+\frac{1}{2}(n+3)(n+4)F_n\right).$$

Note that the sequences $\left\{F_n^{(s)}\right\}_{s=1}^{\infty}$ represented in OEIS by the following numbers: for $n=2$ - A000027, $n=3$ - A000096, $n=4$ - A006503, $n=5$ - A006504, etc.

Now, let us differentiate identity (16)

$$\frac{2^s}{s!}s\left(1-z^2\right)^{s-1}(-2z)U_{n+s-1}^{(s)}(z)+\frac{2^s}{s!}\left(1-z^2\right)^{s}U_{n+s-1}^{(s+1)}(z)=$$
$$=(-1)^s\sum_{j=0}^{s}(-1)^j\binom{n+s-1-j}{n-1}\binom{n+2s}{j}U_{n+2s-1-2j}^{(1)}(z),$$

and substitute $z=i/2$. Then, applying formula (11), we came out with

$$4s5^{s-1}F_n^{(s)}+2(s+1)5^sF_{n-1}^{(s+1)} =$$
$$=2\sum_{j=0}^{s}\binom{n+2s}{j}\binom{n+s-1-j}{n-1}F_{n+2s-2j-1}^{(1)},$$

whence

$$F_n^{(s)} = -\frac{2(s-1)}{5s}F_{n+1}^{(s-1)} +$$
$$+\frac{1}{5^{s-1}s}\sum_{j=0}^{s-1}\binom{n+2s-1}{j}\binom{n+s-1-j}{n}F_{n+2s-2j-2}^{(1)}. \quad (19)$$

Formula (19) allows obtaining various corollaries. For example, let $s=2$:

$$F_n^{(2)} = \frac{1}{10}\left((n+1)F_{n+2}^{(1)}-2F_{n+1}^{(1)}+(n+3)F_n^{(1)}\right).$$

By taking into account (18), formula (19) takes the form

$$F_n^{(s)} = \frac{1}{5^{s-1}s}\sum_{j=0}^{s-1}\binom{n+2s-1}{j}\binom{n+s-1-j}{n}\cdot$$
$$\cdot\left(F_{n+2s-2j-2}^{(1)}-\frac{2(s-1)}{5}F_{n+2s-2j-1}\right).$$

Using (19) it is not difficult to obtain other identities that connect convolved numbers *Fibonacci*.

For k -sections of the *Fibonacci* sequence the *Binet* formulas are obvious. From formula (11) for $s=0$ and $z=\begin{cases}(i/2)L_k, k-odd,\\ (1/2)L_k, k-even\end{cases}$ formula (3) follows immediately. Note, that formula (10) is a consequence of formulas (7), (13) for $s=0$ and $z=-\varphi^{2k}$.

For convolved k -sections of the *Fibonacci* sequence $\Phi_{n,k}^{(s)}$, the generating function is defined similarly to the case $F_n^{(s)}$:

$$f_k^{(s)}(z)=\frac{z}{\left(1-L_kz+(-1)^kz^2\right)^{s+1}};$$
$$\frac{z}{\left(1-L_kz+(-1)^kz^2\right)^{s+1}}=\sum_{j=1}^{\infty}\Phi_{j,k}^{(s)}z^j.$$

Indeed, using [7, p.216], [11], we obtain

$$\frac{z}{1-L_kz+(-1)^kz^2}\cdot\frac{1}{1-L_kz+(-1)^kz^2}=$$
$$=\sum_{j=1}^{\infty}\left(\sum_{l=0}^{n-1}\Phi_{l+1,k}\Phi_{n-l,k}\right)z^j=\sum_{j=1}^{\infty}\Phi_{j,k}^{(1)}z^j,$$
$$\frac{z}{\left(1-L_kz+(-1)^kz^2\right)^s}\cdot\frac{1}{1-L_kz+(-1)^kz^2}=$$
$$=\sum_{j=1}^{\infty}\left(\sum_{l=0}^{n-1}\Phi_{l+1,k}\Phi_{n-l,k}^{(s-1)}\right)z^j=\sum_{j=1}^{\infty}\Phi_{j,k}^{(s)}z^j.$$

Whence

$$\Phi_{n,k}^{(s)}=\frac{1}{2^ss!}\begin{cases}(-i)^{n-1}U_{n+s-1}^{(s)}\left(\frac{i}{2}L_k\right), \; k-odd,\\ U_{n+s-1}^{(s)}\left(\frac{1}{2}L_k\right), \; k-even\end{cases}. \quad (20)$$

Binet type formulas for convolved numbers $\Phi_{n,k}^{(s)}$ are obtained from (14), (15), (20) by substituting $-\varphi^{2k}$ for z if k is odd, and φ^{2k} if k is even:

$$\Phi_{n,k}^{(s)} = (-1)^{k(n-1)}\varphi^{k(-n+1)} \cdot$$
$$\cdot \sum_{j=0}^{n-1}(-1)^{jk}\binom{n+s-1-j}{s}\binom{s+j}{s}\varphi^{2jk},$$
$$\Phi_{n,k}^{(s)} = \frac{\varphi^{-k(n+2s)}}{\left(\varphi^{k}+\varphi^{-k}\right)^{2s+1}} \cdot$$
$$\cdot \sum_{j=0}^{s}((-1)^{(k-1)j}\binom{n+2s}{j}\binom{n+s-1-j}{n-1} \cdot .$$
$$\cdot\left(-(-1)^{kn}\varphi^{2kj}+\varphi^{2k(n+2s-j)}\right))$$

From (16) and (6) the following formula is obtained, which is a complete analogy of formula (18):

$$\Phi_{n,k}^{(s)} = 5^{-s}\left(F_k\right)^{-2s-1} \cdot$$
$$\cdot \sum_{j=0}^{s}(-1)^{(k-1)j}\binom{n+2s}{j}\binom{n+s-1-j}{n-1}F_{k(n+2s-2j)} . \quad (21)$$

An equivalent formula is

$$\Phi_{n,k}^{(s)} = 5^{-s}\left(F_k\right)^{-2s} \cdot$$
$$\cdot \sum_{j=0}^{s}(-1)^{(k-1)j}\binom{n+2s}{j}\binom{n+s-1-j}{n-1}\Phi_{(n+2s-2j),k} .$$

In particular cases $s=1$, $s=2$ we have:

$$\Phi_{n,k}^{(1)} = \frac{1}{5\left(F_k\right)^2}\left(n\Phi_{n+2,k}+(-1)^{k-1}(n+2)\Phi_{n,k}\right);$$
$$\Phi_{n,k}^{(2)} = \frac{1}{25\left(F_k\right)^4}\left(\frac{1}{2}n(n+1)\Phi_{n+4,k}+\right.$$
$$\left.+(-1)^{k-1}n(n+4)\Phi_{n+2,k}+\frac{1}{2}(n+3)(n+4)\Phi_{n,k}\right) .$$

Note that $\Phi_{3,1}^{(s)} = \frac{1}{2}(s+1)(s+4)$, $\Phi_{3,2}^{(s)} = \frac{1}{2}(s+1)(9s+16)$; the sequence $\left\{\Phi_{3,3}^{(s)}\right\}_{s=1}^{\infty} = \{50, 99, 164, 245, 342, \ldots\}$ has a special name "number of walks on a cubic lattice" [5], its OEIS number is A005570.

The sequences $\left\{\Phi_{n,k}^{(s)}\right\}_{n=1}^{\infty}$ for $k=3,4,\ldots$, $s=1,2,\ldots$ are not represented in the OEIS encyclopedia [5]. For the sequence

$$\left\{\Phi_{n,3}^{(1)}\right\}_{n=1}^{\infty} = \{1, 8, 50, 280, 1475, 7472, 36836, \ldots\},$$

the corresponding screenshot is presented in Fig. 1. Similar results are obtained for all sequences $\left\{\Phi_{n,k}^{(s)}\right\}_{n=1}^{\infty}$, $k=3,4,\ldots$, $s=1,2,\ldots$. It is interesting to note that even the sequences $\left\{\Phi_{n,k}\right\}_{n=1}^{\infty}$, $k=13,14,15,\ \ldots$, are not represented in the OEIS.

It follows from (10), (13), (21) that

$$\Phi_{n,k}^{(s)} = \sum_{j=0}^{\left[\frac{n-1}{2}\right]}(-1)^{(k-1)j}\binom{n+s-1-j}{j}\binom{n+s-1-2j}{s}\left(L_k\right)^{n-1-2j} .$$

Then

$$\sum_{j=0}^{\left[\frac{n-1}{2}\right]}(-1)^{(k-1)j}\binom{n+s-1-j}{j}\binom{n+s-1-2j}{s}\left(L_k\right)^{n-1-2j} =$$
$$= 5^{-s}\left(F_k\right)^{-2s-1}\sum_{j=0}^{s}(-1)^{(k-1)j}\binom{n+2s}{j}\binom{n+s-1-j}{n-1}F_{k(n+2s-2j)} . \quad (22)$$

Description of the sequence $\left\{\Phi_{n,k}^{(s)}\right\}_{n=1}^{\infty}$ for the OEIS:

– by definition

$$\Phi_{n,k} = F_{nk}/F_k ,\quad \Phi_{n,k}^{(1)} = \sum_{j=0}^{n-1}\Phi_{j+1,k}\Phi_{n-j,k} ,$$
$$\Phi_{n,k}^{(s)} = \sum_{j=0}^{n-1}\Phi_{j+1,k}\Phi_{n-j,k}^{(s-1)} ,\quad s=2,3,\ldots ;$$

– via generating function $\Phi_{n,k}^{(s)}$ are the coefficients in the series expansion of the generating function

$$f(z) = \frac{z}{\left(1-L_k z+(-1)^k z^2\right)^{s+1}} ;$$

– by *Binet* formula

$$\Phi_{n,k}^{(s)} = \frac{\varphi^{-k(n+2s)}}{\left(\varphi^{k}+\varphi^{-k}\right)^{2s+1}}\sum_{j=0}^{s}((-1)^{(k-1)j}\binom{n+2s}{j} \cdot$$
$$\cdot\binom{n+s-1-j}{n-1}\left(-(-1)^{kn}\varphi^{2kj}+\varphi^{2k(n+2s-j)}\right)) ;$$

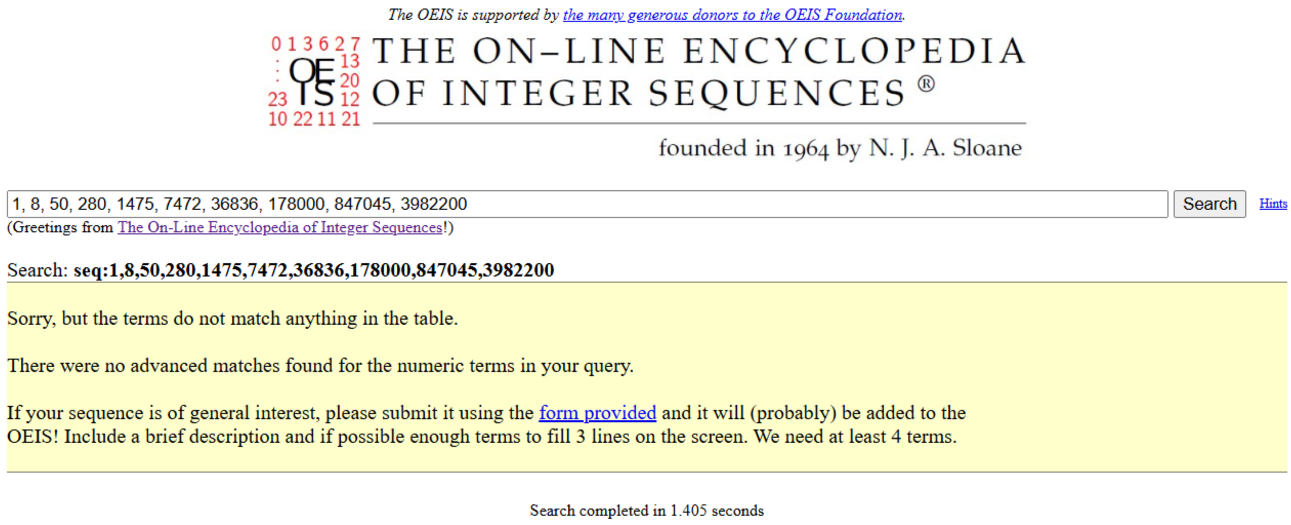

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Fig.1. **Screenshot of checking for the sequence** $\left\{\Phi_{n,3}^{(1)}\right\}_{n=1}^{\infty}$

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– the explicit formula via *Fibonacci* numbers

$$\Phi_{n,k}^{(s)}=5^{-s}\left(F_k\right)^{-2s-1}\sum_{j=0}^{s}\left(\left(-1\right)^{(k-1)j}\binom{n+2s}{j}\cdot\binom{n+s-1-j}{n-1}F_{k(n+2s-2j)}\right);$$

– via recurrence sequence:

define the linear form $\left(1-L_k z+\left(-1\right)^k z^2\right)^{s+1}=\Lambda\left(1,z,z^2,\ldots,z^{2s+2}\right)$; the recurrence sequence is:

$$\Lambda\left(x_{n+2s+2},x_{n+2s+1},\ldots,x_n\right)=0,\ x_j=\Phi_{j,k}^{(s)},\ j=1,\ldots,n+2s+1.$$

SOME MORE IDENTITIES

Identities connecting *Fibonacci* numbers, *Lucas* numbers, binomial numbers have been known for a long time, and this community is replenished almost every day [7], [15], [16], [18], [19]. In this section, consequences of the identities obtained below will be presented. Of course, most of these identities are known (or easily derived from known ones), but, nevertheless, we hope that some identities are new.

From formulas (4) we obtain that

$$\sum_{j=0}^{n-1}F_{j+1}F_{n-j}=\frac{1}{5}\left(nF_{n+2}+(n+2)F_n\right),$$

$$\sum_{j=0}^{n-1}F_{j+1}\left((n-j)F_{n+2-j}+(n+2-j)F_{n-j}\right)=$$
$$=\frac{1}{5}\left(\frac{1}{2}n\left(n+1\right)F_{n+4}+n\left(n+4\right)F_{n+2}+\frac{1}{2}\left(n+3\right)\left(n+4\right)F_n\right).$$

Using the formula from example 18.7 [7, p.222], we obtain the consequence

$$\sum_{j=1}^{[n/2]}j\binom{n-j}{j}=\frac{1}{5}\left((n-1)F_{n+1}+(n+1)F_{n-1}\right)=$$
$$=\frac{1}{5}\left(nL_n-F_n\right).$$

From (18), taking into account that $F_1^{(s)}=1$, $F_2^{(s)}=s+1$, $F_3^{(s)}=\frac{1}{2}\left(s+1\right)\left(s+4\right)$, $F_4^{(s)}=\frac{1}{6}\left(s+1\right)\left(s+2\right)\left(s+9\right)$, $F_5^{(s)}=\frac{1}{24}\left(s+1\right)\left(s+2\right)\left(s+4\right)\left(s+15\right)$, it follows that

$$\frac{1}{5^s}\sum_{j=0}^{s}\binom{1+2s}{j}F_{1+2s-2j}=1,$$

$$\frac{1}{5^s}\sum_{j=0}^{s}\binom{2+2s}{j}\left(1+s-j\right)F_{2+2s-2j}=s+1,$$

$$\frac{1}{5^s}\sum_{j=0}^{s}\binom{3+2s}{j}\binom{2+s-j}{2}F_{3+2s-2j}=\frac{1}{2}\left(s+1\right)\left(s+4\right),$$

$$\frac{1}{5^s}\sum_{j=0}^{s}\binom{4+2s}{j}\binom{3+s-j}{3}F_{4+2s-2j}=\frac{1}{6}\left(s+1\right)\left(s+2\right)\left(s+9\right),$$

$$\frac{1}{5^s}\sum_{j=0}^{s}\binom{5+2s}{j}\binom{4+s-j}{4}F_{5+2s-2j}= \\ =\frac{1}{24}(s+1)(s+2)(s+4)(s+15),$$

etc. (everywhere s is natural number).

From the definition of numbers $\Phi_{n,k}^{(s)}$ (formula (5)) we obtain:

$$\sum_{j=0}^{n-1}F_{(j+1)k}F_{(n-j)k}= \\ =\frac{1}{5F_k}\left(nF_{(n+2)k}+(-1)^{k-1}(n+2)F_{nk}\right),$$

$$\sum_{j=0}^{n-1}F_{(j+1)k}\left((n-j)F_{(n-j+2)k}+(-1)^{k-1}(n-j+2)F_{(n-j)k}\right)= \\ =\frac{1}{5F_k}(\frac{1}{2}n(n+1)F_{(n+4)k}+(-1)^{k-1}n(n+4)F_{(n+2)k}+ \\ +\frac{1}{2}(n+3)(n+4)F_{nk}).$$

Let us take into account that $\Phi_{1,k}^{(s)}=1$, $\Phi_{2,k}^{(s)}=(s+1)L_k$, $\Phi_{3,k}^{(s)}=\frac{1}{2}(s+1)(s+2)L_{2k}+(s+1)^2(-1)^k$. Then

$$\Phi_{4,k}^{(s)}=\frac{1}{2}(s+1)(s+2)(s+3)(L_k)^3- \\ -(s+1)(s+2)(-1)^k L_k$$

(by virtue of equalities

$$\frac{1}{2^s s!}U_{2+s}^{(s)}(z)=2(s+1)(s+2)z^2-(s+1),$$

$$\frac{1}{2^s s!}U_{3+s}^{(s)}(z)=\frac{4}{3}(s+1)(s+2)(s+3)z^3- \\ -2(s+1)(s+2)z.$$

Then

$$5^{-s}(F_k)^{-2s-1}\sum_{j=0}^{s}(-1)^{(k-1)j}\binom{1+2s}{j}F_{k(1+2s-2j)}=1, \quad (23)$$

or

$$(F_k)^{2s+1}=5^{-s}\sum_{j=0}^{s}(-1)^{(k-1)j}\binom{1+2s}{j}F_{k(1+2s-2j)},$$

$$5^{-s}(F_k)^{-2s-1}\cdot \\ \cdot\sum_{j=0}^{s}(-1)^{(k-1)j}\binom{2+2s}{j}(1+s-j)F_{k(2+2s-2j)}=, \quad (24) \\ =(s+1)L_k$$

or

$$(F_k)^{2s}=5^{-s}\frac{1}{(s+1)F_{2k}}\cdot \\ \cdot\sum_{j=0}^{s}(-1)^{(k-1)j}\binom{2+2s}{j}(1+s-j)F_{k(2+2s-2j)},$$

$$5^{-s}(F_k)^{-2s-1}\cdot \\ \cdot\sum_{j=0}^{s}(-1)^{(k-1)j}\binom{3+2s}{j}\binom{2+s-j}{2}F_{k(3+2s-2j)}=, \quad (25) \\ =\frac{1}{2}(s+1)(s+2)L_{2k}+(s+1)^2(-1)^k$$

$$5^{-s}(F_k)^{-2s-1}\sum_{j=0}^{s}(-1)^{(k-1)j}\binom{4+2s}{j}\binom{3+s-j}{3}F_{k(4+2s-2j)}= \\ =\frac{1}{6}(s+1)(s+2)(s+3)(L_k)^3-(s+1)(s+2)(-1)^k L_k. \quad (26)$$

For small values of k or s the formulas can be obtained, which, as a rule, have already been discovered earlier (most likely by other methods). For example, from (23), (24) taking into account that $L_k=F_{2k}/F_k$ (for $s=1, 2, 3$), we get

$$F_{2k}(F_k)^2=\frac{1}{5}\left(F_{4k}+(-1)^{k-1}2F_{2k}\right),$$

$$F_{2k}(F_k)^4=\frac{1}{25}\left(F_{6k}+(-1)^{k-1}4F_{4k}+5F_{2k}\right),$$

$$F_{2k}(F_k)^6=\frac{1}{125}\cdot \\ \cdot\left(F_{8k}+(-1)^{k-1}6F_{6k}+14F_{4k}+(-1)^{k-1}14F_{2k}\right),$$

$$L_k=\frac{F_{4k}-(-1)^k 2F_{2k}}{F_{3k}-(-1)^k 3F_k}=\frac{F_{6k}-(-1)^k 4F_{4k}+5F_{2k}}{F_{5k}-(-1)^k 5F_{3k}+10F_k},$$

etc.

From (25) for $s=0, 1, 2, 3$:

$$\frac{F_{3k}}{F_k}=L_{2k}+(-1)^k,$$

$$\frac{1}{5(F_k)^3}\left(3F_{5k}+(-1)^{k-1}5F_{3k}\right)=3L_{2k}+4(-1)^k,$$

$$\frac{1}{25(F_k)^5}\left(2F_{7k}+(-1)^{k-1}7F_{5k}+7F_{3k}\right)=2L_{2k}+3(-1)^k,$$

$$\frac{1}{125(F_k)^7}\left(5F_{9k}+(-1)^{k-1}27F_{7k}+54F_{5k}+(-1)^{k-1}42F_{3k}\right)= \\ =5L_{2k}+8(-1)^k,$$

etc.

From (26) for $s = 0, 1, 2, 3$:

$$\frac{F_{4k}}{F_k} = (L_k)^3 - (-1)^k 2L_k ,$$

$$\frac{1}{5(F_k)^3}\left(2F_{6k} + (-1)^{k-1} 3F_{4k}\right) = 2(L_k)^3 - (-1)^k 3L_k ,$$

$$\frac{1}{25(F_k)^5}\left(5F_{8k} + (-1)^{k-1} 16F_{8k} + 14F_{4k}\right) = \\ = 5(L_k)^3 - (-1)^k 6L_k ,$$

$$\frac{1}{125(F_k)^7}\left(F_{10k} + (-1)^{k-1} 5F_{8k} + 9F_{6k} + (-1)^{k-1} 6F_{4k}\right) = \\ = (L_k)^3 - (-1)^k L_k ,$$

etc.

Using formula (22), one can also obtain further identities for different values of n, k, s.

CONCLUSION

This paper considers a further generalization of Fibonacci numbers, namely, folded k-section numbers of the Fibonacci sequence. The proposed generalization of Fibonacci numbers is based on the relationships between the derivatives of Chebyshev polynomials of the second kind and the Chebyshev polynomials themselves, as well as on the relationships between the folded k-section numbers of the Fibonacci sequence and the derivatives of Chebyshev polynomials of the second kind via Lucas numbers. A family of generalized sequences with higher order growth and more complex coupling coefficients is obtained, making it ideal for sparse data compression. It is shown that higher-order derivatives of Chebyshev polynomials have proven effective as a basis for solving certain problems in number theory, namely, constructing new sequences. It is noted that the role of Chebyshev polynomials is difficult to overestimate; they represent a powerful tool for solving a variety of applied problems: interpolation theory, approximation theory, numerical analysis, dynamical systems theory, number theory, etc. However, derivatives of Chebyshev polynomials, especially higher-order ones, are encountered significantly less frequently in the literature, primarily in studies of the general properties of orthogonal polynomials. This article, to some extent, fills this gap. It is also proven that the resulting sequences are original and not presented in the OEIS encyclopedia, confirming the originality of the proposed approach to constructing Fibonacci sequences using Chebyshev polynomials.

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Згорнуті числа *k*- перетину послідовності Фібоначчі: властивості, наслідки

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АНОТАЦІЯ

При вирішенні безлічі прикладних завдань у галузі комп'ютерних наук, що використовують теорію інтерполяції, теорію наближення, чисельний аналіз, теорію динамічних систем, теорію чисел використовують багаточлени Чебишева. Однак при формуванні псевдовипадкових рекурентних послідовностей використання похідних багаточленів Чебишева, особливо вищих порядків, зустрічається в літературі значно рідше, хоча **питання є достатньо актуальним.** Ця стаття певною мірою заповнює цю прогалину. Показано, що серед рекурентних послідовностей, які використовуються для аналізу інформації та підвищення її криптозахисту найбільш популярні послідовність Фібоначчі та її узагальнення. **Метою дослідження є** подальше узагальнення чисел Фібоначчі, а саме, згорнуті числа *k*-перетину послідовності Фібоначчі. В процесі дослідження використано сучасні **методи теорії чисел**. Проведено визначення властивостей отриманих послідовностей, знаходження нових зв'язків між їх елементами. Запропоновано подальший розвиток послідовностей типу

Фібоначчі, який заснований на зв'язку між похідними багаточленів Чебишева другого роду та власне поліномами Чебишева, а також на зв'язку згорнутих чисел *k*-перетину послідовності Фібоначчі з похідними поліномів Чебишева другого роду через числа Лукаса. Отримано ряд тотожностей, що пов'язують числа Фібоначчі та числа Люка. Показано, що похідні багаточленів Чебишева вищого порядку виявилися ефективним базисом на вирішення деяких завдань теорії чисел, саме, побудови нових послідовностей. Таким чином, **в результаті дослідження** отримано сімейство узагальнених послідовностей зі зростанням вищого порядку і складнішими коефіцієнтами зв'язку ніж у відомих послідовностей Фібоначчі. Це робить отримані узагальнені послідовності ідеальними для стиснення розріджених даних, вирішення низки завдань у сфері захисту. Доведено, що отримані послідовності є оригінальними і представлені в енциклопедії OEIS, що підтверджує потенціал запропонованого підходу до формування різних послідовностей, які можуть бути використані для підвищення надійності інформаційних систем.

Ключові слова: інформаційні системи; шифрування даних; алгоритм лінійного зсуву; числа Фібоначчі; числа Лукаса; складені числа Фібоначчі; k-перерізи послідовності Фібоначчі; похідні поліномів Чебишева другого роду

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