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Spatial synchronization of cellular automata in evolutionary processes simulation tasks

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ABSTRACT

Many applied tasks are simulated by difference equations that describe the vector of system states evolution in time. However it is required to take into account the spatial structure of simulated processes or systems in some tasks. In paper the possibility of a spatio-temporal processes simulation by cellular automata is considered. The brief review of two-dimensional cellular automata properties is provided. The principle of the most famous two-dimensional cellular automata "Game of Life" is described. Also the general way to set these automata in an analytical form by Reaction-Diffusion equation is considered. Concrete forms of the Reaction equation and Diffusion equation are constructed and invariant sets for this system are defined. The generalization of analytical cellular automata representation in total is provided. As an example, the model of population development is considered. It utilizes the classic Ferhulst equation, in which the spatial structure is taken into account having form of the cumulative neighbors' impact on population changes rate. As per using of analytical form of cellular automata, different schemas of system spatio-temporal characteristics control are suggested. These schemas are based on feedback: delayed feedback (that is one that uses previous system states) and predictive feedback (that is one that uses predicted system states). As a result there is managed to synchronize spatial configuration of cellular automata and it can be interpreted as stable population development. Particularly, cellular automata could work in cycle with cycle length set earlier. For cellular automata evolution visualization the algorithms and their computer implementation are developed. Discrepancy function is suggested, due to which it is possible to evaluate the synchronization accuracy. Research results and examples of received configurations are presented.

Keywords: cellular automata; dynamical systems; chaos; cycle stabilization; predictive control; synchronization

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INTRODUCTION

Many tasks are simulated due to discrete dynamical systems in Physics, Biological Science, Chemistry, Economics, Social Science, etc. [1]. In such systems the state is described by a vector that evolves in time and is considered as discrete one. Vector evolution in phase space does not depend on others vectors evolution, but just on itself vector state in previous time moments. However the spatial structure of system model is necessary to be taken into account in many cases. To resolve this issue cellular automata (CA) are often been utilized as model.

Cellular automata are a specific case of discrete dynamical system. Cellular automata are widely used for processes and systems simulation. Thus in [2-4] due to CA traffic flows are simulated, in [5-10] epidemic spreading, CA are used for GIS system simulation in [11-14] etc. Traditionally, cells' state value is restricted by finite set of states, and cells' transition rules for changing from one's state one to another one by logical functions. However, the description and building of complicated enough systems are difficult. Also such CA setting does not allow build control systems. To eliminate pointed problems CA are set in analytical form.

Nonlinear discrete dynamical systems, even those one that are not taken into account with a spatial structure, often have a complex behavior. Such systems can have chaotic attractors that contain countable number of non-stable cycles. One of the ways of these attractors investigation is their skeletons' constructing through periodic orbits with long enough periods.

In [15-17] search of periodic orbits methods are developed with a set period. They are based on feedback principle, due to which unknown periodic orbits with set period length are stabilized. The delayed feedback and predictive one are applied. In the delayed feedback information of vector state in current time moment and previous ones is used. In predictive control information of vector current state and predicted one is used. Predicted state of a vector is a dynamical system vector state that will be received in some future time moments without control.

To set CA in analytical form, let's choose vectors that are an aggregate of states' vectors of each CA cell as a state space (phase space). And let's represent the operator for transforming the phase space into itself in form of a superposition of linear mapping (Diffusion equation) and nonlinear one (Reaction equation). It is possible to apply all control schemas of discrete systems developed earlier to the received system. If the goal of control is a stabilization of periodic orbits with the set period, then the each CA cell state will be a periodic vector function or function that is close to periodic one. It means that CA spatial structure becomes synchronized, or CA behaves in a way as a connected system. However, visually (visualization is a graphical mapping of a dynamic of corresponding vector components' of the each cell state) if vector function period is big enough, it will be difficult to define spatial synchronization existence. In other words, the impression is made CA works chaotically.

THE PURPOSE OF THE PAPER

The development and application CA spatial synchronization methods are a purpose. For this goal CA is set in an analytical form of discrete dynamical system representation with further building of control systems, which are in form of a feedback, delayed or predictive ones.

FORMULATION OF THE PROBLEM

Cellular automata are a discrete dynamical system that is represented as an aggregate of similar cells that are connected with each other in the similar way. Cells constitute the CA lattice. Each cell is a finite automaton, state of which is defined by cell neighbors' state and itself state. To implement CA the arrays for CA states' storage are provided for current, previous and predictive states. The lattice cells' transition function is defined. Then transition function of states of cells' in time is defined.

The task is to represent CA in form:

$$X_{n+1} = F(X_n), \tag{1}$$

where $X_n = \begin{pmatrix} x_1(n) \\ \vdots \\ x_K(n) \end{pmatrix}$ is a vector that set a cells' state and has size K;

K – is a general number of CA cells.

Elements $x_j(n)$ can be as numbers as vectors.

In [20] to build system (1) the superposition of linear and nonlinear transformation known in literature as Reaction-Diffusion equation (R-D equation) was used [21-23].

In this case system (1) can be presented in form:

$$X_{n+1} = \Phi(DX_n), \tag{2}$$

where $\Phi = \begin{pmatrix} \varphi_1 \\ \vdots \\ \varphi_K \end{pmatrix}$ is a vector function that describe cells' states changes; $i = \overline{1, K}$;

D is a matrix of weight coefficients:

$$\begin{pmatrix} \delta_9 & 0 & \cdots & 0 & \delta_1 & \delta_2 & \delta_3 & \delta_4 & \delta_5 & \delta_6 & \delta_7 & \delta_8 \\ \delta_8 & \delta_9 & 0 & \cdots & 0 & \delta_1 & \delta_2 & \delta_3 & \delta_4 & \delta_5 & \delta_6 & \delta_7 \\ & & & & & \cdots & & & & & & \\ \delta_1 & \delta_2 & \delta_3 & \delta_4 & \delta_5 & \delta_6 & \delta_7 & \delta_8 & \delta_9 & 0 & \cdots & 0 \end{pmatrix}.$$

For edge cells usually the neighbors account is carried out by one of three ways: the periodical continuation, the mirror mapping and the setting of edge conditions. In this paper first way of neighbors' account is used. For central cells an account of neighbors' impact is made by Moore or von Neumann rules [1; 4; 18-19].

Diffusion equation has a form:

$$\begin{cases} y_i = \sum_{s=1}^{r} \delta_s x_{i+s}(n) \\ i = \overline{1, K} \end{cases}, \tag{3}$$

where $r = 9$ is a number of cells are taken into account (current cell and its neighbors) or:

$$Y_n = DX_n, \tag{4}$$

r has not necessary to be equal 9, and it could be a lot of rules of neighbors account.

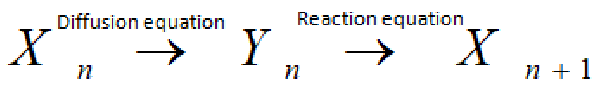

Fig. 1. **The schema of R-D eqation**
Source: **compiled by the author**

Thus if to denote $Y_n = \begin{pmatrix} y_1(n) \\ \vdots \\ y_K(n) \end{pmatrix}$,

and $X_n = \begin{pmatrix} x_1(n) \\ \vdots \\ x_K(n) \end{pmatrix}$, then equations (2-3) will take a form

$$\begin{cases} x_i(n+1) = \varphi_i(y_i(n)) \\ i = \overline{1, K} \end{cases}. \tag{5}$$

The second part of problem is to synchronize CA cells' states using analytical form of CA setting (1) or (5). For example, in such way so the each cell

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evolution would be periodic evolution or close to periodic one.

Wherein the control system is set in form:

$$X_{n+1} = F(X_n) + U_n, \quad (6)$$

where control U_n depends on previous states $X_n, X_{n-1}, \cdots, X_{n-N}$ or on predicted ones that are calculated without control $X_{n+1}, X_{n+2}, \cdots, X_{n+N}$.

Control properties will be described below.

The third part of problem is a visualization of CA behavior that simulates concrete processes in real time schedule as without a control as with a control.

ANALYTICAL REPRESENTATION OF TWO-DIMENSIONAL CELLULAR AUTOMATA "GAME OF LIFE" AND GENERALIZATION

"Game of Life" is the most popular CA among two-dimensional CA [18-19]. The principle of his work lies in each cell of CA takes two states: (one) 1 and (zero) 0. Graphically cells that have state 1 are denoted as black, 0 – as white (Fig. 2).

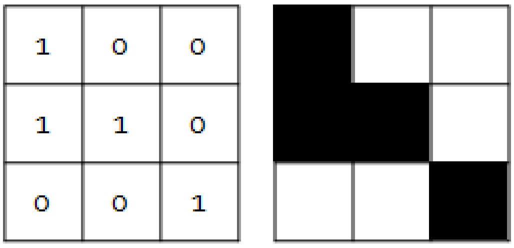

Fig. 2. **Two-dimensional CA presented in digital form and graphical one**
Source: **compiled by the author**

Rules of a transition cell from one state to another one.

1. Life "arise" in empty cell if it has three "alive" neighbors in existing eight ones: $x_9 = 0 \to 1$ (Fig. 3).

2. The cell continues to "live" if it has exactly two or three "alive" neighbors: $x_9 = 1 \to 1$.

3. The cell "dies" if it has less than two neighbors (loneliness) or more than three ones (overpopulation) $x_9 = 1 \to 0$.

The utilization of logical rules provided above makes CA investigation difficult because of a lot of branches and restricts possibilities of automata control. Besides if a set of possible cell values consists of more than two elements than logical rules become boundless. In other cases of more complicated sets of possible states the situation becomes even more difficult. That's why the problem of CA representation in analytical form is important. It becomes possible to study CA, which cells take different state number, up to infinite one.

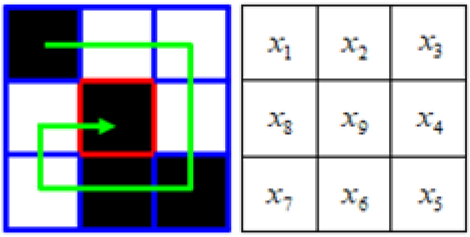

Fig.3. **Cells' location:**
red – current cell;
blue – neighbors, cells' numeration is going through green arrow
Source: **compiled by the author**

Let's represent "Game of Life" CA in analytical form.

To make this let's consider the cell and its neighbors not in form of a lattice but in strip form, where $x_9(n)$ is a cell state in time moment n, $x_j(n),\ j = \overline{1,8}$ is and its neighbors state (Fig. 4).

$x_1(n)$	$x_2(n)$	$x_3(n)$	$x_4(n)$	$x_5(n)$	$x_6(n)$	$x_7(n)$	$x_8(n)$	$x_9(n)$

Fig.4. **The cell and its neighbors presented in form of a strip**
Source: **compiled by the author**

Theorem 1. For "Game of Life" CA weight coefficients of Diffusion equation (3) can be chosen in next way

$$d = \left\{ \frac{2}{17}, \frac{2}{17}, \frac{2}{17}, \frac{2}{17}, \frac{2}{17}, \frac{2}{17}, \frac{2}{17}, \frac{2}{17}, \frac{1}{17} \right\}, \quad (7)$$

and a function in Reaction equation (2):

$$\varphi_1 = \varphi_2 = \cdots = \varphi_K = \varphi,$$

$$\varphi(y) = \begin{cases} 17y - 4, & \frac{4}{17} \le y \le \frac{5}{17} \\ 1, & \frac{5}{17} < y \le \frac{7}{17} \\ -17y + 8, & \frac{7}{17} < y \le \frac{8}{17} \\ 0, & elsewise \end{cases}. \quad (8)$$

Proof. Let's note that y_i can take values from set $\left\{ \frac{i}{17},\ i = 0,1,\ldots,17 \right\}$. If the current cell value equals a one (1) then it will stay a one when and only when $y_i = \frac{2}{17} + \frac{2}{17} + \frac{1}{17} = \frac{5}{17}$, $y_i = \frac{2}{17} + \frac{2}{17} + \frac{2}{17} + \frac{1}{17} = \frac{7}{17}$.

If the cell value equals zero (0) then in the next step it will become a one (1) when and only when $y_i = \frac{2}{17} + \frac{2}{17} + \frac{2}{17} = \frac{6}{17}$. With all of others values of y_i the cell value becomes zero.

Therefore it is necessary to choose the Reaction function $\varphi(y)$ from conditions:

$$\varphi(y) = \begin{cases} 1, \ y \in \left\{\frac{5}{17}, \frac{6}{17}, \frac{7}{17}\right\} \\ 0, \ y \notin \left\{\frac{5}{17}, \frac{6}{17}, \frac{7}{17}\right\} \end{cases}. \tag{9}$$

Function (8) satisfies these conditions. The theorem is proved.

Let's note that coefficients (7) and functions (8) can be chosen not in the only one way, for example,

$$d = \left\{\frac{4}{33}, \frac{4}{33}, \frac{4}{33}, \frac{4}{33}, \frac{4}{33}, \frac{4}{33}, \frac{4}{33}, \frac{4}{33}, \frac{1}{33}\right\}, \quad \text{and}$$

$$\varphi(y) = \begin{cases} 1, \ y \in \left\{\frac{9}{33}, \frac{10}{33}, \frac{13}{33}\right\} \\ 0, \ y \notin \left\{\frac{9}{33}, \frac{10}{33}, \frac{13}{33}\right\} \end{cases}$$

Let's choose Reaction function in form of (8) as more simple continuous piecewise linear function that satisfies conditions (9).

Besides function (8) along with sets A={0,1}, A=[0,1], leaves next set as invariant

$$A = \left\{\frac{i}{n}, \ i = 0,1,\ldots,n\right\}. \tag{10}$$

Theorem 2. The set (10) is invariant for R-D system with Diffusion equation parameters (7) and Reaction equation functions (8).

Proof. Obviously that $y_i = \frac{p_i}{17n}$, where p_i is a positive integer from interval $[0, 17n]$.

Then $\varphi(y_i) = 1$, if $\frac{p_i}{17n} \in \left[\frac{5}{17}, \frac{7}{17}\right]$, $\varphi(y_i) = 0$,

if $\frac{p_i}{17n} \in \left[0, \frac{4}{17}\right] \cup \left[\frac{8}{17}, 1\right]$, $\varphi(y_i) = 17\frac{p_i}{17n} - 4 = \frac{p_i - 4n}{n}$,

if $\frac{p_i}{17n} \in \left[\frac{4}{17}, \frac{5}{17}\right]$.

As $0 \le p_i - 4n \le n$ so $\varphi(y_i) \in A$. Similarly

if $\frac{p_i}{17n} \in \left[\frac{7}{17}, \frac{8}{17}\right]$, then $\varphi(y_i) = \frac{-p_i + 8n}{n} \in A$.

The Theorem is proved.

Thus if the Reaction function is chosen in form of (8) then the arbitrary number of intermediate cells' states in "Game of Life" CA can be taken into account.

GENERAL CASE OF CA SETTING WITH THE SCALAR CELL STATE

Consider the cell state is described by one parameter. Let's denote $x_{ij}(n)$ as cell state where i, j are cell coordinates on CA and n is a discrete time moment.

Let's consider the system that describes transition of the cell from one state to another one:

$$x_{ij}(n+1) = f(x_{ij}(n), y_{ij}(n)), \tag{11}$$

$$y_{ij}(n) = \sum_{k,l=-1}^{1} \mu_{kl}^{(ij)} x_{i+k,j+l}(n), \tag{12}$$

where: $x_{ij}(n)$ is a cell state; $y_{ij}(n)$ is a weighted neighbors' impact; $\sum_{k,l=-1}^{1} \mu_{kl}^{(ij)} = 1$; $\mu_{kl}^{(ij)} \ge 0$.

Usually, μ_{00}^{ij} is supposed to equal 0. If $\mu_{-1-1}^{ij} = \mu_{-11}^{ij} = \mu_{1-1}^{ij} = \mu_{11}^{ij} = 0$ then to take into account neighbors von Neumann rules are used. But if $\mu_{-1-1}^{ij} = \mu_{-11}^{ij} = \mu_{1-1}^{ij} = \mu_{11}^{ij} \ne 0$ then to take into account neighbors Moore rules are used.

Note that for each cell it can be set its own set of weight coefficients that depend on current cell location on space of (i, j). In "Game of Life" CA a neighbors' cells' impact is on an even footing. It means it does not depend on current cell coordinates.

It is clear that system (12) contains (3) as particular case.

THE POPULATION DEVELOPMENT MODEL

Let's consider the known model of population development by Ferhulst:

$$f(x) = 4x(1-x). \tag{13}$$

Using (11) it is possible to construct model (13) with space structure account:

$$x_{n+1} = 4(1 - \alpha y_n) x_n (1 - x_n), \tag{14}$$

where: $\alpha \in [0,1]$ is a coefficient of the neighbors influence intensity; y_n is a function that is defined by formula (11) and means aggregate impact of neighbors on population development rate of the current cell.

The function (12) can be nonlinear and is chosen in dependence on a concrete formulation of the problem.

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In such model each cell has quasi-stochastic behavior that is similar to one, which is described by logistic equation [1].

Such way of system representation lets apply methods of discrete dynamical systems control.

Let's construct system (14) supposing that $\alpha = 0.05$ and visualize results.

CELLULAR AUTOMATA CONTROL. DELAYED FEEDBACK CONTROL

Different systems of the discrete dynamical systems control are considered in works [15-17]. In this paper we apply control methods to CA systems.

Let's consider the nonlinear control that is set by delayed feedback principle, using the information of a CA state in previous steps:

$$x_{ij}(n+1) = f\left(\sum_{k=1}^{N} \varepsilon_k x_{ij}(n-(k-1)T), \sum_{k=1}^{N} \varepsilon_k y_{ij}(n-(k-1)T)\right), \quad (15)$$

where: N is a prehistory length; ε_k are control coefficients; $\varepsilon_k \in [0,1]$; $\sum_{k=0}^{N} \varepsilon_k = 1$; T is a period.

Semi-linear control has a form:

$$x_{ij}(n+1) = (1-\gamma) f\left(\sum_{k=1}^{N} \varepsilon_k x_{ij}(n-(k-1)T), \sum_{k=1}^{N} \varepsilon_k y_{ij}(n-(k-1)T)\right) + \gamma \sum_{k=1}^{N} \tilde{\varepsilon}_k x_{ij}(n-kT+1), \quad (16)$$

$\gamma \in [0,1)$ is a control parameter.

As an example, let's build the semi-linear control with delayed feedback to stabilize cycle of the 8 length. Optimal coefficients ε_i are calculated by known algorithms given in [13]. For visualization, constructing and processing of configurations' data Visual Studio Professional 2019 development environment was used, the application was developed by the object oriented programming language C#.

Initial configuration is chosen in random way. Illustrations, which are presented in Fig. 5, show the CA state in each cycle moment on a corresponding iteration that is pointed under a corresponding picture.

Each cell state is described by value from interval [0, 1]. A grey color shade corresponds to this value, moreover white color corresponds to zero and black color corresponds to one.

Obviously that configuration on iteration 1664 is repeated after 8 steps and the discrepancy function tends to zero $\alpha_{1671} = 5 \times 10^{-8}$ (Fig. 6).

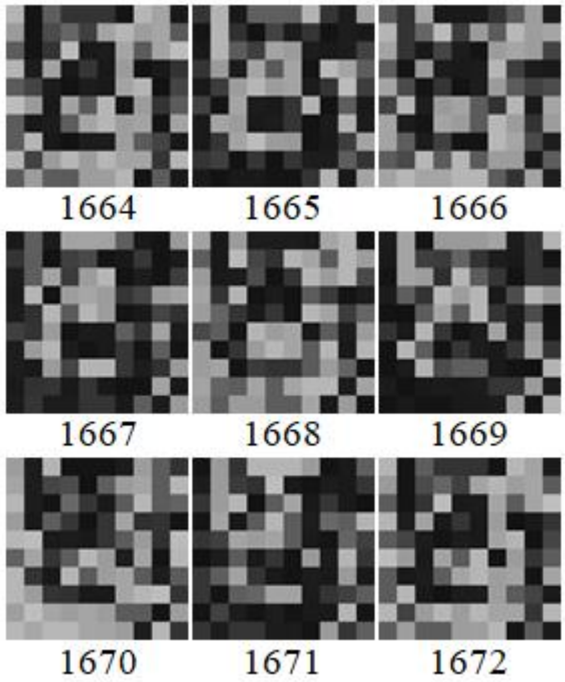

Fig.5. **Cellular automata control by nonlinear control, 8-cycle construction**

***Source:* compiled by the author**

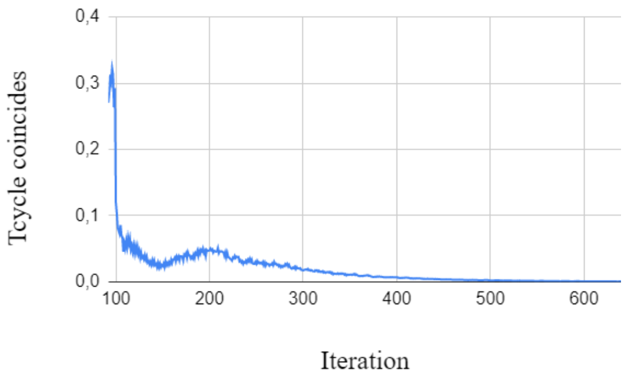

Fig. 6. **Discrepancy function for constructed cellular automata (8-cycle)**

***Source:* compiled by the author**

Visually CA looks like if system behaves chaotically. That's why the values' discrepancy on each step n was provided to show that applied control methods work correctly:

$$\alpha_n = \frac{1}{M \cdot N} \sum_{i=1}^{M} \sum_{j=1}^{N} \left| x_{ij}(n) - x_{ij}(n-T) \right|, \quad (17)$$

where: $M \times N$ is a size of CA; n is an iteration number; T is a cycle; $n > T$.

This discrepancy should tend to zero.

PREDICTIVE CONTROL

In predictive control the predictive cells' values are used while CA works without a control.

In the simplest case such control system has a form:

$$x_{ij}(n+1) = f\left(\frac{\theta}{1+\theta} x_{ij}(n) + \frac{1}{1+\theta} f^{(T)}\left(x_{ij}(n)\right)\right), \quad (18)$$

where: θ is chosen by schema represented in [2]; T is a cycle.

Let's construct the system with the applying of predictive control. The CA size is 10×10, $\theta = 4.2 \times 10^{7}$.

The result of the synchronization of a spatial CA configuration, which cells have a cyclical behavior with period 30, can be evaluated by discrepancy function $\alpha_{2888} = 1 \times 10^{-28}$ (Fig. 7).

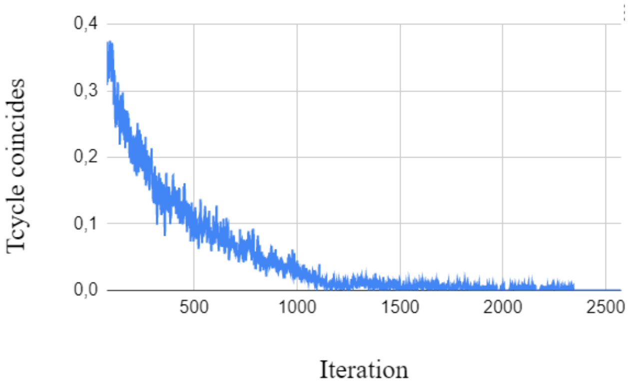

Fig. 7. **Discrepancy function for constructed cellular automata (30-cycle)**
Source: **compiled by the author**

CONCLUSIONS

On the example of the well-known CA "Game of Life" it is shown that CA constructed by logical rules can be represented in analytical form. It allowed each cell of CA take any value from set interval and not only from discrete set $\{0,1\}$ as in classic case. Besides the possibility of the developed schemas of discrete dynamical systems control applying is appeared.

The population development model was considered in conditions of intraspecific competition, i.e. with spatial structure account. Delayed control and predictive one were applied to constructed system. It led to spatial CA synchronization that can be interpreted as stable population development.

In this work was not considered examples with vector value of cell state. Typical representative of such CA are spatio-temporal models of epidemic spreading dynamics.

The system construction in CA form, which allows control epidemics, is a further research direction.

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Просторова синхронізація клітинних автоматів в задачах моделювання еволюційних процесів та керування ними

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АНОТАЦІЯ

Більшість прикладних задач моделюються за допомогою різницевих рівнянь, що описують еволюцію вектору станів систем із плином часу. Проте в деяких задачах необхідно також враховувати просторову структуру процесів та систем, які моделюються. У статті розглядається можливість моделювання просторово-часових процесів за допомогою клітинних автоматів. Наведено короткий огляд властивостей двовимірних клітинних автоматів. Описано принцип роботи найвідомішого двовимірного клітинного автомату «Гра Життя», а також розглянуто узагальнений спосіб побудови цього автомату в аналітичній формі за допомогою рівняння Реакції-Дифузії. Побудовані конкретні форми рівняння Реакції та рівняння Дифузії, та визначені інваріантні множини для цієї системи. Наведено узагальнення аналітичного виду клітинного автомату в цілому. В якості прикладу булла розглянута модель розвитку популяції, що використовує класичне рівняння Ферхюльста, в якій просторова структура враховується у вигляді сукупного впливу сусідів на швидкість зміни популяції. Використовуючи аналітичну форму клітинного автомату, були запропоновані різні схеми контролю просторово-часовими характеристиками системи. Ці схеми базуються на принципі зворотного зв'язку: з запізненням (тобто, що використовує попередні стани системи) та з прогнозом (що використовує передбаченні стани системи). В результаті вдавалось синхронізувати просторову конфігурацію клітинного автомату, що можна інтерпретирувати як стійкий розвиток популяції. Зокрема, клітинний автомат міг функціонувати циклічно із заздалегідь заданною довжиною циклу. Для візуалізації еволюції автоматів були розроблені алгоритми та їх комп'ютерна реалізація. Була запропонована функція нев'язки, по якій можна оцінювати точність синхронізації. В результаті дослідження побудовано приклади отриманих конфігурацій.

Ключові слова: клітинні автомати; динамічні системи; хаос; стабілізація циклів; предикативний контроль; синхронізація

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Пространственная синхронизация клеточных автоматов в задачах моделирования эволюционных процессов и управления ими

АННОТАЦИЯ

Многие прикладные задачи моделируются с помощью разностных уравнений, которые описывают эволюцию вектора состояний системы с течением времени. Однако в некоторых задачах требуется также учитывать пространственную структуру моделируемых процессов или систем. В статье рассматривается возможность моделирования пространственно-временных процессов с помощью клеточных автоматов. Приведен краткий обзор свойств двумерных клеточных автоматов. Описан принцип работы известнейшего двумерного клеточного автомата «Игра Жизнь», а также рассмотрен обобщенный способ задания этого автомата в аналитической форме с помощью уравнения Реакции-Диффузии. Построены конкретные формы уравнения Реакции и уравнения Диффузии, и определены инвариантные множества для этой системы. Приводится обобщение аналитического представления клеточного автомата в целом. В качестве примера рассмотрена модель развития популяции, использующая классическое уравнение Ферхюльста, в которой пространственная структура учитывается в виде совокупного влияния соседей на скорость изменения популяции. Используя аналитическую форму клеточного автомата, были предложены различные схемы управления пространственно-временными характеристиками системы. Эти схемы основаны на принципе обратной связи: запаздывающей (то есть, использующей предыдущие состояния системы) и

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прогнозной (использующей предсказанные состояния системы). В результате удавалось синхронизировать пространственную конфигурацию клеточного автомата, что можно интерпретировать как устойчивое развитие популяции. В частности, клеточный автомат мог функционировать циклически с заранее заданной длиной цикла. Для визуализации эволюции автоматов были разработаны алгоритмы и их компьютерная реализация. Была предложена функция невязки, по которой можно оценивать точность синхронизации. Представлены результаты исследования и примеры полученных конфигураций.

Ключевые слова: клеточные автоматы; динамические системы; хаос; стабилизация циклов; предикативный контроль; синхронизация

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